

## Thermal Aspects of ABJM theory: Currents and Condensations

Dongsu Bak<sup>a</sup> and Sangheon Yun<sup>b</sup>

*a) Physics Department, University of Seoul, Seoul 130-743 Korea*

*b) Korea Institute for Advanced Study, Hoegiro 87, Dondaemun-gu, Seoul 151-747 Korea*

dsbak@uos.ac.kr, sanhan@kias.re.kr

### ABSTRACT

To study thermal aspects of the ABJM theory in the strongly coupled regime, we carry out the  $\mathbb{CP}_3$  invariant dimensional reduction of the type IIA supergravity down to four dimensions. We then investigate zero and finite temperature responses of the operators which are dual to the AdS scalar and vector fields. Two scalar operators are shown to have finite-temperature condensations by coupling of constant source term. The currents dual to the massless and massive gauge fields are not induced by coupling of constant boundary vector potential, which implies that the phase described by black brane background is not superconducting. We also discuss a generalization to charged (dyonic) black holes.

# 1 Introduction

There has been remarkable progress in understanding of the AdS/CFT correspondence between string theories and conformal field theories[1]. The type IIB string theory on  $\text{AdS}_5 \times S^5$  is dual to the four dimensional  $\mathcal{N} = 4$  super Yang-Mills theory[1], which is by now tested to a convincing level. Thus one may now apply this duality to explore some new physics or to gain some precise understanding of strongly coupled regime of field theories where our direct field-theoretic understanding is limited. The Janus correspondence as a controlled deformation of  $\text{AdS}_5/\text{CFT}_4$ [2] is one such example, which leads to interesting predictions for the interface conformal field theories[3]. It also serves as an excellent toy model for understanding of the strongly coupled aspects of the real world QCD at finite temperature. The Debye correlation length and the thermalization time scale of finite temperature  $\mathcal{N} = 4$  super Yang-Mills theory in the strongly coupled regime are examples of such toy application[4, 5].

The type IIA counterpart is proposed recently, in which the ABJM theory is three dimensional  $\mathcal{N} = 6$   $\text{U(N)} \times \overline{\text{U(N)}}$  superconformal Chern-Simons theory with level  $(k, -k)$  and proposed to be dual to the type IIA superstring theory on  $\text{AdS}_4 \times \mathbb{CP}_3$  background[6]. Some test of this new duality has been carried out based on the integrabilities with indication of some additional structure[7, 8]. It is based on the large  $N$  limit where one is taking  $N, k \rightarrow \infty$  while holding the 't Hooft coupling  $\lambda = \frac{N}{k}$  to be fixed. If  $\lambda$  is small, the superconformal Chern-Simons theory can be studied by perturbative analysis while, for strongly coupled regime of large  $\lambda$ , the supergravity is a valid description.

In this note, using the supergravity description, we shall study thermal aspects of the ABJM field theory in the strongly coupled regime. In order to identify the low energy dynamics, we first carry out the consistent,  $\mathbb{CP}_3$  invariant compactification of the type IIA supergravity down to four dimensions. (See also Ref. [9] for a related M theory dimensional reduction, in which extra modes are present compared to its type IIA counterpart.) The resulting potential is minimized at the  $\text{AdS}_4$  vacuum and depends on three bulk scalars which are dual to operators of dimension 4, 5 and 6. Using this 4d action, one may consistently study full nonlinear effects without being limited to just small fluctuations.

In addition the theory involves two bulk  $\text{U}(1)$  gauge fields satisfying Maxwell type equations. One combination is the usual massless  $\text{U}(1)$  gauge field which implies existence of global  $\text{U}(1)$  current of dimension 2. In the field theory side, one has so called di-baryon charge[6] which is always accompanied by magnetic flux due to Gauss law constraint of Chern-Simons theories. Turning on the corresponding global charge will lead to a charged AdS black hole which is necessarily deformed due to the nontrivial dependence on the scalars. The other combination of bulk gauge fields turns out to be massive due to the Higgs mechanism where one scalar that is 4d hodge-dual to the NS-NS three form gets absorbed into gauge degrees. This

massive gauge field is dual to the dimension 5 current operator in the boundary field theory side.

In this note, to understand the low energy dynamics of the ABJM system, we study the response of these operators at zero and finite temperatures considering  $\text{AdS}_4$  and the black brane backgrounds respectively.

We shall demonstrate that, at zero temperature, none of these operators develops a non-zero expectation value under a constant source term. We also compute exact expressions for AC conductivity of the two currents at zero temperature. At finite temperature phase, the dimension 4 and 5 scalar operators possess nonvanishing expectation values meaning condensations. The magnitudes of condensation are proportional to source term and we shall compute these proportionality constants.

The vector dynamics is particularly interesting. The source term coupled to boundary current operators is interpreted as a boundary vector potential. If the presence of this boundary vector potential implies nonvanishing boundary current, this is precisely the superconducting current which is proportional to the vector potential[10, 11]. The system is then in a superconducting phase. For finite temperatures, we first show that the supercurrent exists for the current of general dimensions with exceptions of  $\Delta = 3n \pm 1$  ( $n \in \mathbb{N}$ ). The  $\Delta = 2, 5$  cases belong to this exceptional category and, hence, there exists no superconducting current for our black brane background.

The paper is organized as follows. Section 2 is a brief description of the type IIA supergravity and the  $\text{AdS}_4 \times \mathbb{CP}_3$  backgrounds. In Section 3, we carry out the  $\mathbb{CP}_3$  invariant dimensional reduction down to four dimensions. We pay a particular attention to the consistency of ansatz with the original supergravity equations of motion. For completeness, we also include the dimensional reduction for the skew-whiffed background[12] corresponding to the anti-D2 branes instead of the D2 branes of the ABJM background. Section 4 describes the scalar dynamics and finite temperature condensation of dual field theory operators. Section 5 describes the vector dynamics. We find that both dimension 2 and 5 currents are not superconducting currents. Last section is devoted to concluding remarks and further directions of study.

## 2 Type IIA supergravity and the $\text{AdS}_4 \times \mathbb{CP}_3$ background

We begin our discussion from the type IIA supergravity description of the ABJM field theory. The supergravity description is valid in the limit  $N \rightarrow \infty$  with the 't Hooft coupling  $\lambda = \frac{N}{k}$  kept fixed and large, i.e.  $\lambda \gg 1$ . Hence corresponding dual CFT is necessarily strongly coupled. Of course the small  $\lambda$  region can be studied by direct perturbative analysis of the ABJM field theory. The ABJM theory possesses  $SO(3,2) \times SU(4)$  global bosonic symmetries. The 4d conformal group of  $SO(3,2)$  corresponds to the global symmetry of  $\text{AdS}_4$  spacetime. The

Chern-Simons theory also possesses  $SU(4)$  R-symmetry, which is the symmetry of the  $\mathbb{CP}_3$  directions. Their supersymmetric completion is described by the supergroup of  $Osp(4|6)$ . The Chern-Simons theory has one additional global  $U(1)$  symmetry, which we shall discuss later on. We also consider finite temperature version of this field theory in the large  $\lambda$  regime, which can be described by the IIA supergravity in the  $AdS_4$  black brane background.

In this section, we shall briefly review the type IIA supergravity and some relevant thermodynamic properties of the  $AdS_4$  black brane background. The bosonic part of the Einstein frame type IIA supergravity is described by the action,

$$S_{IIA} = S_{NS} + S_R + S_{CS} \quad (2.1)$$

where

$$\begin{aligned} S_{NS} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R(g) - \frac{1}{2} \nabla_m \phi \nabla^m \phi - \frac{1}{12} e^{-\phi} H_{mnp} H^{mnp} \right] \\ S_R &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ -\frac{1}{4} e^{\frac{3}{2}\phi} F_{mn} F^{mn} - \frac{1}{48} e^{\frac{1}{2}\phi} \tilde{F}_{mnpq} \tilde{F}^{mnpq} \right] \\ S_{CS} &= \frac{1}{2\kappa_{10}^2} \int \left[ \frac{1}{2} B_{[2]} \wedge F_{[4]} \wedge F_{[4]} \right]. \end{aligned} \quad (2.2)$$

The 10d gravity coupling in this action is given by  $2k_{10}^2 = (2\pi)^7 \ell_s^8 g_s^2$  where  $g_s$  denotes the string coupling. We define the dilaton field  $\phi$  as its nonzero mode by subtracting its constant part  $\hat{\phi}$  related to the string coupling  $g_s = e^{\hat{\phi}}$ . The gauge invariant four form field strength  $\tilde{F}_{[4]}$  is defined by

$$\tilde{F}_{[4]} = dA_{[3]} + dB_{[2]} \wedge A_{[1]}, \quad (2.3)$$

and the NS-NS three form field strength by  $H_{[3]} = dB_{[2]}$ . The string frame metric is given by the transformation  $g_{mn}^s = e^{\frac{1}{2}\phi} g_{mn}$ , but we shall not use it in this note.

The supergravity equations read

$$\begin{aligned} R_{mn} &= \frac{1}{2} \nabla_m \phi \nabla_n \phi + \frac{1}{2} e^{\frac{3\phi}{2}} \left[ F_{mp} F_n{}^p - \frac{1}{16} g_{mn} F_{pq} F^{pq} \right] \\ &+ \frac{1}{4} e^{-\phi} \left[ H_{mpq} H_n{}^{pq} - \frac{1}{12} g_{mn} H_{pqr} H^{pqr} \right] \\ &+ \frac{1}{12} e^{\frac{\phi}{2}} \left[ \tilde{F}_{mpqr} \tilde{F}_n{}^{pqr} - \frac{3}{32} g_{mn} \tilde{F}_{pqrs} \tilde{F}^{pqrs} \right], \end{aligned} \quad (2.4)$$

$$96 \square \phi = 36 e^{\frac{3}{2}\phi} F_{mn} F^{mn} - 8 e^{-\phi} H_{mnp} H^{mnp} + e^{\frac{1}{2}\phi} \tilde{F}_{mnpq} \tilde{F}^{mnpq}, \quad (2.5)$$

$$\nabla_m [e^{\frac{3\phi}{2}} F^{mn}] = -\frac{1}{6} e^{\frac{\phi}{2}} \tilde{F}^{npqr} H_{pqr}, \quad (2.6)$$

$$\nabla_m[e^{-\phi}H^{mnp}] + \frac{1}{2}e^{\frac{\phi}{2}}\tilde{F}^{mnpq}F_{mq} = -\frac{1}{1152}\epsilon^{npr_1\cdots r_8}\tilde{F}_{r_1\cdots r_4}\tilde{F}_{r_5\cdots r_8}, \quad (2.7)$$

$$\nabla_m[e^{\frac{\phi}{2}}\tilde{F}^{mnpq}] - \frac{1}{144}\epsilon^{mnpqr_1\cdots r_6}\tilde{F}_{r_1\cdots r_4}H_{mr_5r_6} = 0, \quad (2.8)$$

which agree with those in Ref. [13].

The supergravity background of the ABJM theory is given by[13, 6]

$$\begin{aligned} ds^2 &= R_s^2 \left[ \frac{1}{4} ds^2(\text{AdS}_4) + ds^2(\mathbb{CP}_3) \right] \\ e^{2\hat{\phi}} &= g_s^2 = \frac{R_s^2}{k^2 \ell_s^2} \\ F_{[4]} &= \frac{3}{8} k g_s R_s^2 \hat{\epsilon}_4 \\ F_{[2]} &= k g_s d\omega = 2k g_s J, \end{aligned} \quad (2.9)$$

where the IIA curvature radius  $R_s$  is

$$R_s^2 = 4\pi\sqrt{2\lambda}\ell_s^2. \quad (2.10)$$

The unit-radius AdS Poincaré metric is given by

$$ds^2(\text{AdS}_4) = \frac{1}{z^2} \left[ -dt^2 + dx^2 + dy^2 + dz^2 \right], \quad (2.11)$$

and  $\hat{\epsilon}_4$  denotes its 4d volume form. We parametrize the  $\mathbb{CP}_3$  metric as[15]

$$\begin{aligned} ds^2(\mathbb{CP}_3) &= d\xi^2 + \frac{\sin^2 2\xi}{4} \left( d\psi + \frac{\cos \theta_1}{2} d\phi_1 - \frac{\cos \theta_2}{2} d\phi_2 \right)^2 + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) \\ &+ \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2), \end{aligned} \quad (2.12)$$

where  $0 \leq \phi_i < 2\pi$ ,  $0 \leq \theta_i \leq \pi$ ,  $0 \leq \xi \leq \frac{\pi}{2}$  and  $0 \leq \psi < 2\pi$ . Note that the volume of unit  $\mathbb{CP}_3$  space is given by  $\pi^3/6$ . The unit  $\mathbb{S}^7$  can be presented as a Hopf fibration over  $\mathbb{CP}_3$ ,

$$ds^2(\mathbb{S}^7) = ds^2(\mathbb{CP}_3) + (d\theta_{10} + \omega)^2, \quad (2.13)$$

with  $0 \leq \theta_{10} < 2\pi$ . The one form  $\omega$  is explicitly given by

$$\omega = -\frac{1}{2} \left( \cos 2\xi d\psi + \cos^2 \xi \cos \theta_1 d\phi_1 + \sin^2 \xi \cos \theta_2 d\phi_2 \right). \quad (2.14)$$

Then the Kähler two form  $J = \frac{1}{2}d\omega$  takes the form

$$\begin{aligned} J &= \frac{1}{4} \left( \sin 2\xi d\xi \wedge (2d\psi + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2) \right. \\ &\quad \left. + \cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\phi_1 + \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\phi_2 \right). \end{aligned} \quad (2.15)$$

Below we shall use the following properties of  $J$ :

$$\begin{aligned}\nabla_a J_{bc} &= 0, \\ J_{ab} J^{bc} &= -\delta_a^c, \\ 8J^{ef} &= \varepsilon^{abcdef} J_{ab} J_{cd},\end{aligned}\tag{2.16}$$

where the indices are raised by unit  $\mathbb{CP}_3$  metric.

We also consider the well known black brane solution with a planar symmetry. In this solution, one is replacing the  $\text{AdS}_4$  metric in (2.12) by the black hole metric

$$ds_4^2 = \frac{1}{z^2} \left[ \frac{dz^2}{h(z)} - h(z) dt^2 + dx^2 + dy^2 \right]\tag{2.17}$$

where

$$h(z) = 1 - \left( \frac{z}{z_H} \right)^3.\tag{2.18}$$

This black brane background is dual to the finite temperature version of the ABJM theory. Due to the quantum scale invariance of theory, this finite temperature field theory depends on only one dimensionful parameter which is the temperature  $T$ . Hence the theory possesses only one finite-temperature phase corresponding to high temperature limit. This temperature is identified with the Hawking temperature of the black brane,

$$T = \frac{1}{4\pi} |h'(z_H)| = \frac{3}{4\pi} \frac{1}{z_H}.\tag{2.19}$$

The basic thermodynamic quantities are as follows; The entropy density reads

$$s = \frac{N^2}{3\sqrt{2\lambda}} \frac{1}{z_H^2} = \frac{N^2}{3\sqrt{2\lambda}} \left( \frac{4\pi}{3} \right)^2 T^2.\tag{2.20}$$

The energy density may be evaluated as

$$\mathcal{E} = \frac{N^2}{6\pi\sqrt{2\lambda}} \frac{1}{z_H^3} = \frac{2N^2}{9\sqrt{2\lambda}} \left( \frac{4\pi}{3} \right)^2 T^3,\tag{2.21}$$

and the pressure  $p = \mathcal{E}/2$  as dictated by the conformal symmetry. We see here that the number of effective degrees in the strongly coupled regime is proportional to  $N^2/\sqrt{\lambda}$ . Since the number of degrees in the weakly coupled, small  $\lambda$  region is simply proportional to  $N^2$ , the effective number of degrees is reduced greatly down by the factor of  $\sqrt{\lambda}$ .

In order to simplify notation, we set up our convention as follows. We note that only  $R_s$  dependence remains in the above background while the  $k$  dependence disappears completely. The gravity equations themselves do not involve any explicit  $k$  dependence either. Then we set  $R_s = 1$  taking  $R_s$  as a length unit if necessary.

### 3 Compactification on $\mathbb{CP}_3$

We perform a consistent,  $\mathbb{CP}_3$  invariant dimensional reduction by taking the following ansatz<sup>1</sup>:

$$\begin{aligned}
ds^2 &= \frac{1}{4}e^{-3\sigma}ds_4^2 + e^\sigma ds^2(\mathbb{CP}_3), \\
F_{ab} &= 2J_{ab}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \\
H_{\mu\nu\lambda} &= \frac{3}{2}e^{\phi-6\sigma}\epsilon_{\mu\nu\lambda}{}^\delta(\pm A_\delta - \bar{A}_\delta - \nabla_\delta\psi), \quad H_{\mu ab} = (\partial_\mu\chi)J_{ab}, \\
\tilde{F}_{\mu\nu\lambda\delta} &= -\frac{3}{8}e^{-\frac{\phi}{2}-9\sigma}\epsilon_{\mu\nu\lambda\delta}(\pm 1 + \chi^2), \\
\tilde{F}_{\mu\nu ab} &= e^{-\frac{\phi}{2}-\sigma}{}^*\tilde{F}_{\mu\nu}J_{ab}, \quad \tilde{F}_{abcd} = -2\chi(J \wedge J)_{abcd},
\end{aligned} \tag{3.1}$$

where we use  $\epsilon_{\mu\nu\lambda\delta} = \sqrt{-g}E_{\mu\nu\lambda\delta}$  with a convention  $E_{0123} = -1$  for the numerical totally antisymmetric number  $E_{\mu\nu\lambda\delta}$ . The 4d hodge dual is then defined by  ${}^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}F_{\alpha\beta}$  and  $\tilde{F}_{\mu\nu}$  is simply an antisymmetric tensor field not defined by the vector potential. We shall turn off all the remaining components. The upper signs in the above ansatz and below correspond to the ABJM theory whereas the lower signs are for the skew-whiffed background[12] corresponding to the anti-D2 branes. From [13], one can check that all of  $\mathbb{CP}_3$  invariant modes are included in the above ansatz. Note also that there are no  $\mathbb{CP}_3$  invariant fermionic modes[13].

The resulting equations of motion can be derived from the action

$$\mathcal{L}_4 = \frac{1}{16\pi G_4} (\mathcal{L}_g + \mathcal{L}_c), \tag{3.2}$$

where

$$\begin{aligned}
\mathcal{L}_g &= R_4(g) - \frac{1}{2}(\nabla\phi)^2 - 6(\nabla\sigma)^2 - \frac{3}{2}e^{-\phi-2\sigma}(\nabla\chi)^2 - e^{\frac{3\phi}{2}+3\sigma}F_{\mu\nu}F^{\mu\nu} - 3e^{-\frac{\phi}{2}-\sigma}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \\
&- 18e^{\phi-6\sigma}(\pm A - \bar{A} - \nabla\psi)^2 + 12e^{-4\sigma} - \frac{9}{2}e^{-\frac{\phi}{2}-9\sigma}(\pm 1 + \chi^2)^2 - \frac{3}{2}e^{\frac{3\phi}{2}-5\sigma} - 6\chi^2e^{\frac{\phi}{2}-7\sigma}
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
\mathcal{L}_c &= 6e^{-\frac{\phi}{2}-\sigma}\Lambda^{\mu\nu}\left(\tilde{F}_{\mu\nu} - 2\chi e^{-\frac{\phi}{2}-\sigma}{}^*\tilde{F}_{\mu\nu} - \chi^2F_{\mu\nu} - \partial_\mu\bar{A}_\nu + \partial_\nu\bar{A}_\mu\right) \\
&+ 6\chi e^{-\phi-2\sigma}\tilde{F}_{\mu\nu}{}^*\tilde{F}^{\mu\nu} + 6\chi\tilde{F}^{\mu\nu}({}^*F_{\mu\nu} + 2\chi e^{-\frac{\phi}{2}-\sigma}F_{\mu\nu}) - 4\chi^3F_{\mu\nu}{}^*F^{\mu\nu}.
\end{aligned} \tag{3.4}$$

The  $g_{\mu\nu}$ ,  $\phi$  and  $\sigma$  equations are solely following from the variations of  $\mathcal{L}_g$ . Namely the corresponding variations of  $\mathcal{L}_c$  are vanishing completely. The  $\Lambda^{\mu\nu}$  variation leads to the constraint

$$\tilde{F}_{\mu\nu} - 2\chi e^{-\frac{\phi}{2}-\sigma}{}^*\tilde{F}_{\mu\nu} - \chi^2F_{\mu\nu} - \bar{F}_{\mu\nu} = 0. \tag{3.5}$$

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<sup>1</sup>For the compactification spectrum, see [12]-[16]. See also Ref. [9] for the related dimensional reduction from the M theory whose spectra are different from those of the type IIA theory.

The  $\tilde{F}_{\mu\nu}$  variation determines  $\Lambda_{\mu\nu}$  by

$$\Lambda_{\mu\nu} = \tilde{F}_{\mu\nu} - \chi e^{\frac{\phi}{2} + \sigma} * F_{\mu\nu}, \quad (3.6)$$

and the remaining equations are

$$\begin{aligned} \nabla_\mu [e^{\frac{3\phi}{2} + 3\sigma} F^{\mu\nu}] &= 3(\partial_\mu \chi) * \tilde{F}^{\mu\nu} + 9e^{\phi - 6\sigma} (\pm 1 + \chi^2) (\pm A^\nu - \bar{A}^\nu - \nabla^\nu \psi), \\ \nabla_\mu [e^{-\frac{\phi}{2} - \sigma} \tilde{F}^{\mu\nu}] &= (\partial_\mu \chi) * F^{\mu\nu} - 3e^{\phi - 6\sigma} (\pm A^\nu - \bar{A}^\nu - \nabla^\nu \psi), \\ \nabla_\mu [e^{-\phi - 2\sigma} \nabla^\mu \chi] &= 4\chi e^{\frac{\phi}{2} - 7\sigma} + 6\chi (\pm 1 + \chi^2) e^{-\frac{\phi}{2} - 9\sigma} - 2 * \tilde{F}^{\mu\nu} (F_{\mu\nu} - e^{-\phi - 2\sigma} \tilde{F}_{\mu\nu}). \end{aligned} \quad (3.7)$$

Details of derivation are relegated to Appendix. The constraint (3.5) can be solved in terms of vector potentials by

$$\tilde{F} = \frac{1}{1 + 4\chi^2 e^{-\phi - 2\sigma}} \left[ \bar{F} + \chi^2 F + 2\chi e^{-\frac{\phi}{2} - \sigma} * (\bar{F} + \chi^2 F) \right]. \quad (3.8)$$

For the field theory interpretation, one needs the 4d Newton constant whose value is given by

$$\frac{1}{16\pi G_4} = \frac{N^2}{12\pi\sqrt{2\lambda}}. \quad (3.9)$$

As identified in Ref. [6], the field  $F_{\mu\nu}$  electrically couples to D0 brane current while its magnetic hodge dual couples to D6 branes wrapping  $\mathbb{CP}_3$ . The other field  $\tilde{F}_{\mu\nu}$  couples electrically to D4 branes wrapping  $\mathbb{CP}_2$  cycle and magnetically to D2 wrapping  $\mathbb{CP}_1$  cycle of  $\mathbb{CP}_3$ .

Finally, we note that all the 4d equations are  $R_s$  independent and scale invariant. One may further set the only length scale  $z_H = 1$  in the black brane metric as  $h(z) = 1 - z^3$ . This corresponds to the mass unit  $4\pi T/3$ , which will be recovered whenever needed. Below we shall be only interested in the case of the ABJM background for the further analysis.

## 4 Scalar dynamics

There are three scalars,  $\phi$ ,  $\sigma$  and  $\chi$  for our low energy effective action. Unlike the  $\text{AdS}_5$  case of the type IIB theory, all of them become massive. In this section we shall explore their zero and finite temperature dynamics.

Under  $\phi$  and  $\chi$  changes, the gravity potential in (3.2) becomes infinitely large when  $|\phi|$  and  $|\chi| \rightarrow \infty$ . The potential has an absolute minimum at  $\phi = \sigma = \chi = 0$ , at which the potential takes a value  $-6$ . When  $\sigma \rightarrow -\infty$ , the potential becomes infinitely large as well. In the other limit  $\sigma \rightarrow \infty$ , the potential goes to zero. The latter corresponds to the decompactification limit where the volume of  $\mathbb{CP}_3$  becomes infinitely large.



For the small fluctuations to the linear order, the  $\phi$  and  $\sigma$  fields can be diagonalized by the linear combinations,

$$\phi_+ = \frac{\phi + 18\sigma}{\sqrt{28}}, \quad \phi_- = \frac{\sqrt{3}(3\phi - 2\sigma)}{\sqrt{28}}, \quad (4.1)$$

which lead to the massive scalar equation,

$$\nabla^2 \Phi - m_\Phi^2 \Phi = 0, \quad (4.2)$$

with masses  $m_{\phi_+}^2 = 18$  and  $m_{\phi_-}^2 = 4$  respectively. The  $\chi$  field, on the other hand, is already diagonal with mass  $m_\chi^2 = 10$ . As is well known, scalar field in the  $\text{AdS}_4$  space behaves, near the boundary  $z = 0$ , as[17]

$$\Phi(x, z) = a_\Phi(x) z^{3-\Delta} + b_\Phi(x) z^\Delta + \dots, \quad (4.3)$$

where  $\dots$  denotes all the remaining powers of  $z$  and  $x_\mu$  is for the boundary directions  $x_0, x_1, x_2$ . The number  $\Delta$  in the above expression is given by

$$\Delta = \frac{1}{2} \left( 3 + \sqrt{9 + 4m_\Phi^2} \right), \quad (4.4)$$

which can be identified with the scaling dimension of the field theory operator dual to  $\Phi$ . It follows that the dimensions of the dual operators  $O_{\phi_+}$ ,  $O_{\phi_-}$  and  $O_\chi$  are respectively 6, 4 and 5. This is quite consistent with the fact that these operators should belong to some low-lying protected supermultiplets where dimensions of component operators do not receive any quantum corrections such that their bosonic sector consisting of even number of fields have only integer dimension.

Turning on  $a_\Phi$  in (4.3) in the supergravity side corresponds to turning on the source term in the field theory side,

$$\delta \mathcal{L}_{FT} = a_\Phi(x) O_\Phi(x). \quad (4.5)$$

Then the operator expectation value  $\langle O_\Phi(x) \rangle$  is evaluated as

$$\langle O_\Phi(x) \rangle = \frac{\delta I_{SUGRA}}{\delta a_\Phi} = \frac{3\alpha}{32\pi G_4} b_\Phi(x), \quad (4.6)$$

where  $\alpha$  is an extra factor in the scalar kinetic term  $-(\nabla\Phi)^2/2$  in  $\mathcal{L}_g$  related to the normalizations of our scalar field. The boundary condition for the scalar field at the horizons at  $z = 1$  or  $z = \infty$  ( $T = 0$ ) is

$$h \Phi'|_{z=1, \infty} = 0, \quad (4.7)$$

where prime denotes a derivative with respect to  $z$ . This is the requirement that there should not be any boundary contribution from the horizon when one evaluates the on-shell supergravity action<sup>2</sup>.

Now we consider the massive scalar equation with an ansatz  $\Phi = U(z)$  in order to study the finite temperature condensation of operators. For  $T = 0$ , the scalar equation becomes

$$z^2 U'' - 2z U' - m_\Phi^2 U = 0, \quad (4.8)$$

which leads to the simple solution,

$$U = a_\Phi z^{3-\Delta} + b_\Phi z^\Delta. \quad (4.9)$$

For  $\Delta = 4, 5, 6$  of our problem, the boundary condition at  $z = \infty$  demands  $b_\Phi = 0$ . Hence there is no zero-temperature condensation of operators by coupling of source term in the field theory side.

The equation for  $T \neq 0$  becomes

$$z^4 \left( \frac{h}{z^2} U' \right)' - m_\Phi^2 U = 0. \quad (4.10)$$

This leads to the general solution

$$U = a_\Phi z^{3-\Delta} {}_2F_1\left(\frac{3-\Delta}{3}, \frac{3-\Delta}{3}; \frac{6-2\Delta}{3}; z^3\right) + b_\Phi z^\Delta {}_2F_1\left(\frac{\Delta}{3}, \frac{\Delta}{3}; \frac{2\Delta}{3}; z^3\right), \quad (4.11)$$

where  ${}_2F_1(\alpha, \beta; \gamma; x)$  is the hypergeometric function. For  $\Delta = 6$  of  $m_{\phi_+}^2 = 18$ , the hypergeometric function can be simplified to

$$U = a_{\phi_+} \left( \frac{1}{z^3} - \frac{1}{2} \right) - 6b_{\phi_+} \left( 2 + (2z^{-3} - 1) \ln(1 - z^3) \right). \quad (4.12)$$

The boundary condition at the horizon dictates then

$$\frac{b_\Phi}{a_\Phi} = - \frac{4\Gamma\left(\frac{3}{2} - \frac{\Delta}{3}\right) \Gamma\left(\frac{\Delta}{3}\right)}{4^{\frac{2\Delta}{3}} \Gamma\left(\frac{3-\Delta}{3}\right) \Gamma\left(\frac{1}{2} + \frac{\Delta}{3}\right)} \left( \frac{4\pi}{3} T \right)^{2\Delta-3}, \quad (4.13)$$

where our mass unit is recovered. For  $\Delta = \frac{3}{2} + 3n$  ( $n = 1, 2, \dots$ ), one has  $a_\Phi = b_\Phi = 0$ , which means turning on the operator is not allowed. For the other case  $\Delta = 3n$  ( $n = 1, 2, \dots$ ), one finds  $b_\Phi = 0$ . Hence there is no condensation of the corresponding operators. The  $\phi_+$  fluctuation belongs to this category where  $\Delta = 6$ . The massless case of  $\Delta = 3$  is special; Both  $a_\Phi$  and  $b_\Phi$  are allowed and independent of each other. For the generic mass, the above ratio is nonvanishing.

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<sup>2</sup>In principle,  $\Phi = 0$  boundary condition is allowed but this in general leads to null solution besides some special cases which are not relevant to us.

In particular, the  $\phi_-$  and  $\chi$  fluctuations belong to this category leading to finite-temperature condensations.

For  $m_{\phi_-}^2 = 4$  with  $\Delta = 4$ , one has

$$\begin{aligned}\langle O_{\phi_-} \rangle &= \frac{3}{32\pi G_4} \frac{\sqrt{3} [\Gamma(\frac{1}{3})]^6}{160\pi^3} \left( \frac{4\pi}{3} T \right)^5 a_{\phi_-} = \frac{2\pi\sqrt{6} [\Gamma(\frac{1}{3})]^6}{1215} \frac{N^2 T^5}{\sqrt{\lambda}} a_{\phi_-} \\ &= (4.6823) \frac{N^2 T^5}{\sqrt{\lambda}} a_{\phi_-},\end{aligned}\tag{4.14}$$

where we recover our mass unit. For  $m_\chi^2 = 10$  with  $\Delta = 5$ , one has

$$\begin{aligned}\langle O_\chi \rangle &= -\frac{9}{32\pi G_4} \frac{9\sqrt{3} [\Gamma(\frac{2}{3})]^6}{56\pi^3} \left( \frac{4\pi}{3} T \right)^7 a_\chi = -\frac{128\sqrt{6}\pi^3 [\Gamma(\frac{2}{3})]^6}{567} \frac{N^2 T^7}{\sqrt{\lambda}} a_\chi \\ &= -(105.70) \frac{N^2 T^7}{\sqrt{\lambda}} a_\chi.\end{aligned}\tag{4.15}$$

Therefore at finite temperature, the condensations of  $O_\chi$  and  $O_{\phi_-}$  occur if one dials the corresponding source terms. This also verifies that the  $T \neq 0$  phase of the ABJM theory is distinct from that of  $T = 0$ .

## 5 Vector dynamics and superconducting current

We now turn to the analysis of the vector fields,  $A_\mu$  and  $\bar{A}_\mu$ . At the linear order,  $\tilde{F}_{\mu\nu} \sim \bar{F}_{\mu\nu}$  and the combination,  $A_\mu - \bar{A}_\mu$ , becomes massive due to the Higgs mechanism. The absorbed scalar  $\psi$  is four dimensional hodge dual to the three-form field  $H_{\mu\nu\lambda}$ . Including this combination, the gauge kinetic terms can be diagonalized by the linear combinations,

$$A_\mu^H = \frac{\sqrt{3}}{2}(A_\mu - \bar{A}_\mu), \quad A_\mu^B = \frac{1}{2}(A_\mu + 3\bar{A}_\mu),\tag{5.1}$$

with masses  $m_H^2 = 12$  and  $m_B^2 = 0$ . The massless field  $A_\mu^B$  couples to the di-baryon current  $J^B$  of the ABJM theory. The field equation including Gauss law constraint takes a form,

$$\frac{k}{4\pi} *_3 \text{tr}(F_{\text{U(N)}} + F_{\bar{\text{U(N)}}}) = J_B\tag{5.2}$$

where  $*_3$  denotes three dimensional hodge dual of the boundary theory and  $F_{\text{U(N)}}$  and  $F_{\bar{\text{U(N)}}}$  are the 3d field strengths of  $\text{U(N)}$  and  $\bar{\text{U(N)}}$  gauge fields. Hence the charge of  $J_B$  is always accompanied by magnetic flux, which is an important characteristic of Chern-Simons theories. Since the dual AdS gauge field  $A_\mu^B$  is massless, one may turn on the di-baryon charge and consider charged AdS black hole background. The scalar fields,  $\phi, \sigma$  and  $\chi$ , can be consistently

set to zero leading to a simple charged (dyonic) black hole solution with  $A_\mu = \bar{A}_\mu$ [9]. We shall get back to this issue later on.

To the linear order the gauge field equation satisfies

$$\nabla_\mu F^{\mu\nu} - m_\phi^2 A^\nu = 0. \quad (5.3)$$

If  $m_\phi^2 \neq 0$ , the consistency of the above equation requires  $\nabla_\mu A^\mu = 0$ , which also follows from the scalar equation  $\psi$  in the gauge  $\psi = 0$ . The massless case is special but the treatment is basically the same.

As in the scalar case, the fields, in the near boundary region of  $z = 0$ , behave as

$$A_{\bar{\mu}} = a_{\bar{\mu}}(x) z^{2-\Delta} + b_{\bar{\mu}}(x) z^{\Delta-1} + \dots \quad (5.4)$$

where barred indices are for the boundary spacetime directions, i.e.  $\bar{\mu} = 0, 1, 2$ . The number  $\Delta$ ,

$$\Delta = \frac{1}{2} \left( 3 + \sqrt{1 + 4m_\phi^2} \right), \quad (5.5)$$

is the dimension of current operator dual to bulk gauge field  $A_\mu$ . For our cases of  $m_\phi^2 = 0$  and 12, we find  $\Delta = 2$  and 5 respectively. The dimension two current operator is the di-baryon current  $J_B$  as we already mentioned. The other dimension five current is dual to the massive gauge field, which includes dynamics of the absorbed scalar degree.

We begin with the zero temperature case. With an ansatz  $A_{\bar{\mu}} = \mathcal{A}_{\bar{\mu}}(z)$ , the Maxwell equation becomes

$$z^2 \mathcal{A}_{\bar{\mu}}'' - m_\phi^2 \mathcal{A}_{\bar{\mu}} = 0, \quad (5.6)$$

whose solution is simply

$$\mathcal{A}_{\bar{\mu}} = a_{\bar{\mu}} z^{2-\Delta} + b_{\bar{\mu}} z^{\Delta-1}. \quad (5.7)$$

The vanishing boundary contribution at  $z = \infty$  leads to  $b_{\bar{\mu}} = 0$  for  $\Delta \geq 2$ . Consequently, there is no induced current by coupling of constant vector potential  $a_{\bar{\mu}}$ .

We now turn to AC response of current at zero temperature. For this purpose, we shall consider only transverse current with an ansatz,  $A_y = \mathcal{A}_y(z) e^{-i\omega t + ipx}$ . The Maxwell equation is reduced to

$$z^2 \mathcal{A}_y'' + \left( (\omega^2 - p^2) z^2 - m_\phi^2 \right) \mathcal{A}_y = 0, \quad (5.8)$$

whose general solution is

$$\mathcal{A}_y = C_1 z^{\frac{1}{2}} I_\nu \left( z \sqrt{p^2 - \omega^2} \right) + C_2 z^{\frac{1}{2}} K_\nu \left( z \sqrt{p^2 - \omega^2} \right), \quad (5.9)$$

with  $v = \Delta - \frac{3}{2}$ . For  $p^2 > \omega^2$ ,  $I_v(x)$  becomes exponentially large at the horizon  $z = \infty$  so that one requires  $C_1 = 0$ . If  $p^2 < \omega^2$ , one may use the so-called purely ingoing boundary condition where flux near horizon region should be directed toward the horizon. This again leads to  $C_1 = 0$ . Since  $K_v(x)$  behaves

$$K_v(z) = \frac{\pi}{2 \sin(v\pi)} \left[ \frac{z^{-v}}{2^{-v} \Gamma(1-v)} - \frac{z^v}{2^v \Gamma(1+v)} \right] + \dots \quad (5.10)$$

for the small  $z$ , we conclude

$$\frac{b_y}{a_y} = \frac{\pi (\omega^2 - p^2)^{\Delta - \frac{3}{2}}}{(2i)^{2\Delta-3} \Gamma\left(\Delta - \frac{1}{2}\right) \Gamma\left(\Delta - \frac{3}{2}\right) \sin\left(\Delta - \frac{1}{2}\right) \pi}. \quad (5.11)$$

For the massless case of  $\Delta = 2$ , the AC conductivity for the baryon current is then evaluated as

$$\sigma_B = \frac{J_y}{i\omega a_y} = \frac{1}{8\pi G_4} \frac{b_y}{i\omega a_y} = \frac{N^2}{6\pi\sqrt{2\lambda}} \left(1 - \frac{p^2}{\omega^2}\right)^{\frac{1}{2}}, \quad (5.12)$$

while

$$\sigma_H = \frac{N^2}{9450\pi\sqrt{2\lambda}} \omega^6 \left(1 - \frac{p^2}{\omega^2}\right)^{\frac{7}{2}}. \quad (5.13)$$

The overall dependence on the frequency is basically dictated by the underlying conformal symmetry and dimension of the current operator.

For the finite temperature, we again consider turning on vector potential that is independent of the boundary coordinates. The Lorentz symmetry is now broken due to the non-zero temperature but the rotation and translation symmetries remain. We take an ansatz  $A_{\bar{\mu}} = \mathcal{A}_{\bar{\mu}}(z)$  as in  $T = 0$  case. For  $\mathcal{A}_i$  ( $i = 1, 2$ ), the Maxwell equation becomes

$$z^2 \left( h \mathcal{A}_i' \right)' - m_\phi^2 \mathcal{A}_i = 0. \quad (5.14)$$

While turning on constant vector potential, the existence of the current proportional to the vector potential as  $J_i \propto a_i$  corresponds to superconducting current[11]. Existence of such current implies that the system under consideration is in a superconducting phase. We shall focus on the investigation of this possibility within our dimensionally reduced system<sup>3</sup>.

The general solution of the above equation can be given by

$$\mathcal{A}_i = a_i z^{2-\Delta} {}_2F_1\left(\frac{2-\Delta}{3}, \frac{4-\Delta}{3}; \frac{6-2\Delta}{3}; z^3\right) + b_i z^{\Delta-1} {}_2F_1\left(\frac{\Delta-1}{3}, \frac{\Delta+1}{3}; \frac{2\Delta}{3}; z^3\right), \quad (5.15)$$

in terms of the hypergeometric functions. The boundary condition  $h \mathcal{A}_i' = 0$  at  $z = 1$  leads to the ratio,

$$\frac{b_i}{a_i} = - \frac{\Gamma\left(\frac{6-2\Delta}{3}\right) \Gamma\left(\frac{\Delta-1}{3}\right) \Gamma\left(\frac{\Delta+1}{3}\right)}{\Gamma\left(\frac{2-\Delta}{3}\right) \Gamma\left(\frac{4-\Delta}{3}\right) \Gamma\left(\frac{2\Delta}{3}\right)} \left(\frac{4\pi}{3} T\right)^{2\Delta-3}. \quad (5.16)$$

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<sup>3</sup> For the discussion of superconductivity arising in the more general dimensional reduction, see Ref. [9, 18].

This is in general nonvanishing with some exception specified below and leads to the nonvanishing supercurrent in the presence of constant vector potential in the boundary theory. The exception corresponds to  $\Delta = 3n - 1, 3n + 1$  where  $n$  is a natural number. Interestingly, our cases of  $\Delta = 2, 5$  belong to this exceptional category. Therefore there is no superconducting current implying that our system is not in a superconducting phase. Note also that the hypergeometric functions for  $\Delta = 2, 5$  are greatly simplified and can be given in terms of simple elementary functions. But we shall not present their detailed forms here.

For the time component, the equation is reduced to

$$z^2 h \mathcal{A}_0'' - m_\phi^2 \mathcal{A}_0 = 0. \quad (5.17)$$

whose solution is

$$\mathcal{A}_0 = a_0 z^{2-\Delta} {}_2F_1\left(\frac{2-\Delta}{3}, \frac{1-\Delta}{3}; \frac{6-2\Delta}{3}; z^3\right) + b_0 z^{\Delta-1} {}_2F_1\left(\frac{\Delta-1}{3}, \frac{\Delta-2}{3}; \frac{2\Delta}{3}; z^3\right), \quad (5.18)$$

This is not a valid solution when  $\Delta = 3n$  ( $n \in \mathbb{N}$ ). But we shall not consider  $\Delta = 3n$  since they are not relevant to our analysis. The boundary condition required for the vanishing contribution at the horizon can be identified as  $A_0 = 0$  unless  $\Delta = 2$ . This leads to the ratio

$$\frac{b_0}{a_0} = -\frac{\Gamma\left(\frac{6-2\Delta}{3}\right) \Gamma\left(\frac{\Delta+1}{3}\right) \Gamma\left(\frac{\Delta+2}{3}\right)}{\Gamma\left(\frac{4-\Delta}{3}\right) \Gamma\left(\frac{5-\Delta}{3}\right) \Gamma\left(\frac{2\Delta}{3}\right)} \left(\frac{4\pi}{3} T\right)^{2\Delta-3}. \quad (5.19)$$

The source term  $a_0$  can be related to a chemical potential while  $b_0$  is an induced charge density. Hence non-zero chemical potential induces corresponding charge density unless  $\Delta = 3n + 2, 3n + 1$  ( $n \in \mathbb{N}$ ). For  $\Delta = 3n + 2, 3n + 1$  ( $n \in \mathbb{N}$ ), the ratio vanishes and there is no induced charge density.  $J_H^0$  belongs to this exceptional category. Thus it is not possible to turn on a nonvanishing charge density by adjusting the chemical potential.

Finally for  $\Delta = 2$ , the solution simply becomes

$$A_0 = a_0^B + b_0^B z \quad (5.20)$$

where  $a_0^B$  is a constant which can be shifted away and  $b_0^B$  is the di-baryon charge density which can be freely adjusted. Hence this fluctuation describes the deformation of charged AdS black hole at linear order.

## 6 Conclusions

In this note, we considered the thermal aspects of the ABJM field theories in the strongly coupled regime using the supergravity description. For this purpose, we carry out the  $\mathbb{CP}_3$  invariant

dimensional reduction of type IIA supergravity down to four dimensions. We then study the zero and finite temperature responses of various operators which are dual to bulk scalar and vector fields. We have shown that condensations of dimension 4 and 5 scalar operators occur at finite temperature by coupling of constant source term. The currents, on the other hand, are not induced by coupling of boundary vector potential, which implies that the system is not in a superconducting phase. We have also computed the AC conductivities at zero temperature.

The existence of condensation implies that the finite temperature phase is distinct from the zero temperature phase of the ABJM theory. This is rather obvious since the temperature breaks the scale invariance introducing dimensionful temperature scale. The situation may turn into more interesting case if one dials the global U(1) charge that is dual to the massless bulk gauge field. It is described by the charged AdS black hole, which is known to have the zero temperature limit. Unlike the case of the R-charged AdS<sub>5</sub> black holes[19, 20], the zero temperature phase appears to be thermodynamically stable. Its phase structure and thermodynamic properties are of interest[21]. The relation between charge and energy near zero temperature [22, 23] and the peculiar ground state entropy demand a further understanding of the ABJM system with finite di-baryon charge density. The scalar  $\chi$  and the gauge field  $A^H$ , both are dual to the dimension 5 operators, get mixed up if one turns on (dyonic) charges and may lead to a very nontrivial response properties. These issues are currently under investigation.

The other is on the spatial correlation length scales of the ABJM theory in the strongly coupled regime. As discussed in Ref. [4], the longest correlation is governed by the true mass gap and the Debye mass for the charged excitation is the lowest in the CT-odd sector of the spectrum. Our four dimensional gravity action is particularly suited for this problem. This is also currently under investigation[24].

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## A $\mathbb{CP}_3$ invariant dimensional reduction

In this appendix, we shall explain some key ingredients of the dimensional reduction and show that all the equations of motion may be derived from the four dimensional effective Lagrangian. The starting point is to solve the Bianchi identity,  $d\tilde{F}_{[4]} = -H_{[3]} \wedge F_{[2]}$ . We take the ansatz where  $\tilde{F}_{\mu\nu\lambda a} = \tilde{F}_{\mu abc} = H_{\mu\nu a} = 0$  and  $F_{ab} = 2J_{ab}$ . By setting  $\tilde{F}_{abcd} = -2\chi(J \wedge J)_{abcd}$  together with

$\tilde{F}_{\mu\nu ab} = e^{-\frac{\phi}{2}-\sigma} * \tilde{F}_{\mu\nu} J_{ab}$ , we get

$$H_{\mu ab} = \nabla_\mu \chi J_{ab} \quad \text{and} \quad \nabla_\mu (e^{-\frac{\phi}{2}-\sigma} \tilde{F}^{\mu\nu}) = \nabla_\mu \chi * F^{\mu\nu} - \frac{1}{3} \varepsilon^{\nu\mu\lambda\rho} H_{\mu\lambda\rho}, \quad (\text{A.1})$$

from the Bianchi identity. With a further ansatz,  $\tilde{F}_{\mu\nu\lambda\rho} = X e^{-\frac{\phi}{2}-9\sigma} \varepsilon_{\mu\nu\lambda\rho}$ , the Bianchi identity can be solved by

$$\tilde{F}_{\mu\nu\lambda\rho} = -\frac{3}{8} (\pm 1 + \chi^2) e^{-\frac{\phi}{2}-9\sigma} \varepsilon_{\mu\nu\lambda\rho}. \quad (\text{A.2})$$

Eq. (2.8) for  $\tilde{F}_{\mu\nu ab}$  leads to the constraint equation in (3.5), which can be solved by introducing  $\bar{A}_\mu$  as  $\bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu = \tilde{F}_{\mu\nu} - 2\chi e^{-\frac{\phi}{2}-\sigma} * \tilde{F}_{\mu\nu} - \chi^2 F_{\mu\nu}$ . The 10d equation for  $H_{\mu\nu\lambda}$  is then solved by

$$H_{\mu\nu\lambda} = \frac{3}{2} e^{\phi-6\sigma} \varepsilon_{\mu\nu\lambda}{}^\delta (\pm A_\delta - \bar{A}_\delta - \nabla_\delta \psi). \quad (\text{A.3})$$

Using the Lagrange multiplier  $\Lambda_{\mu\nu}$ , we incorporate the above constraint into the Lagrangian. After straightforward calculation, we come to the dimensionally reduced effective Lagrangian in (3.2). This Lagrangian possesses two kinds of auxiliary fields,  $\tilde{F}_{\mu\nu}$  and  $\Lambda_{\mu\nu}$ . Variation with respect to  $\tilde{F}_{\mu\nu}$  leads to

$$\Lambda_{\mu\nu} = \tilde{F}_{\mu\nu} - \chi e^{\frac{\phi}{2}+\sigma} * F_{\mu\nu}, \quad (\text{A.4})$$

while  $\Lambda_{\mu\nu}$  variation gives us the constraint (3.5). The variations with respect to  $\bar{A}_\nu$  and  $A_\nu$  lead to

$$\nabla_\mu (e^{-\frac{\phi}{2}-\sigma} \Lambda^{\mu\nu}) + 3e^{\phi-6\sigma} (\pm A^\nu - \bar{A}^\nu - \nabla^\nu \psi) = 0, \quad (\text{A.5})$$

and

$$\nabla_\mu (e^{\frac{3\phi}{2}+3\sigma} F^{\mu\nu}) - 3\nabla_\mu \chi * \tilde{F}^{\mu\nu} - 9e^{\phi-6\sigma} (\pm 1 + \chi^2) (\pm A^\nu - \bar{A}^\nu - \nabla^\nu \psi) = 0, \quad (\text{A.6})$$

respectively. The equations of motion for  $\phi$  and  $\sigma$  following from the effective Lagrangian read

$$\begin{aligned} \nabla^2 \phi = & -\frac{3}{2} e^{-\phi-2\sigma} (\nabla \chi)^2 + \frac{9}{4} e^{\frac{3\phi}{2}-5\sigma} + 3\chi^2 e^{\frac{\phi}{2}-7\sigma} - \frac{9}{4} e^{-\frac{\phi}{2}-9\sigma} (\pm 1 + \chi^2)^2 \\ & + \frac{3}{2} e^{\frac{3\phi}{2}+3\sigma} F_{\mu\nu} F^{\mu\nu} + 18e^{\phi-6\sigma} (\pm A - \bar{A} - \nabla \psi)^2 - \frac{3}{2} e^{-\frac{\phi}{2}-\sigma} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} 4\nabla^2 \sigma = & -e^{-\phi-2\sigma} (\nabla \chi)^2 + 16e^{-4\sigma} - \frac{5}{2} e^{\frac{3\phi}{2}-5\sigma} - 14\chi^2 e^{\frac{\phi}{2}-7\sigma} - \frac{27}{2} e^{-\frac{\phi}{2}-9\sigma} (\pm 1 + \chi^2)^2 \\ & + e^{\frac{3\phi}{2}+3\sigma} F_{\mu\nu} F^{\mu\nu} - 36e^{\phi-6\sigma} (\pm A - \bar{A} - \nabla \psi)^2 - e^{-\frac{\phi}{2}-\sigma} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \end{aligned} \quad (\text{A.8})$$



which are consistent with the 10d equations. From the metric variation, one has

$$\begin{aligned}
R_{\mu\nu} = & \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi + 6\nabla_\mu\sigma\nabla_\nu\sigma + \frac{3}{2}e^{-\phi-2\sigma}\nabla_\mu\chi\nabla_\nu\chi + 2e^{\frac{3\phi}{2}+3\sigma}(F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}) \\
& + 6e^{-\frac{\phi}{2}-\sigma}(\tilde{F}_{\mu\alpha}\tilde{F}_\nu^\alpha - \frac{1}{4}g_{\mu\nu}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta}) + 18e^{\phi-6\sigma}(\pm A_\mu - \bar{A}_\mu - \nabla_\mu\psi)(\pm A_\nu - \bar{A}_\nu - \nabla_\nu\psi) \\
& - \frac{1}{2}g_{\mu\nu}\left[12e^{-4\sigma} - \frac{3}{2}e^{\frac{3\phi}{2}-5\sigma} - 6\chi^2e^{\frac{\phi}{2}-7\sigma} - \frac{9}{2}e^{-\frac{\phi}{2}-9\sigma}(\pm 1 + \chi^2)^2\right], \tag{A.9}
\end{aligned}$$

which is consistent with the 10d Einstein equation in (2.4). Note that one needs to impose  $\tilde{F}_{\mu\nu}$  constraint before taking the metric variation. Namely, one has to plug the constraint,  $\Lambda^{\mu\nu} = \tilde{F}^{\mu\nu} - \chi e^{\frac{\phi}{2}+\sigma} * F^{\mu\nu}$ , into the action because the metric variation of  $\sqrt{-g}\Lambda^{\mu\nu}$  which involves both  $\sqrt{-g}\tilde{F}^{\mu\nu}$  and  $\sqrt{-g}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$  is not well defined before imposing constraint.

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